

Find the derivative for each of the following, by applying FTOC.

1. $g(x) = \int_1^x (t^2 - 1)^{20} dt$

2. $g(x) = \int_{-1}^x \sqrt{t^3 + 1} dt$

$$\begin{aligned} g'(x) &= ((x)^2 - 1)^{20}(1) - ((1)^2 - 1)^{20}(0) \\ g'(x) &= (x^2 - 1)^{20} \end{aligned}$$

$$\begin{aligned} g'(x) &= \sqrt{(x)^3 + 1}(1) - \sqrt{(-1)^3 + 1}(0) \\ g'(x) &= \sqrt{x^3 + 1} \end{aligned}$$

3. $g(u) = \int_{\pi}^u \frac{1}{1+t^4} dt$

$$\begin{aligned} g'(u) &= \frac{1}{1+(u)^4}(1) - \frac{1}{1+(\pi)^4}(0) \\ g'(u) &= \frac{1}{1+u^4} \end{aligned}$$

4. $f(x) = \int_x^2 \cos(t^2) dt$

$$\begin{aligned} f'(x) &= \cos((2)^2)(0) - \cos((x)^2)(1) \\ f'(x) &= -\cos(x^2) \end{aligned}$$

5. $h(x) = \int_2^{x^{-1}} \sin^4 t dt$

$$h(x) = \int_2^{x^{-1}} \sin^4 t dt$$

$$h'(x) = \sin^4(x^{-1})(-x^{-2}) - \sin^4(2)(0)$$

$$h'(x) = -\frac{\sin^4(\frac{1}{x})}{x^2}$$

6. $g(x) = \int_1^{\sqrt{x}} \frac{s^2}{s^2 + 1} ds$

$$\begin{aligned} g(x) &= \int_1^{x^{\frac{1}{2}}} \frac{s^2}{s^2 + 1} ds \\ g'(x) &= \frac{\left(\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}\right)^2}{\left(\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}\right)^2 + 1} \left(\frac{1}{2}x^{-\frac{1}{2}}\right) - \frac{(1)^2}{(1)^2 + 1}(0) \\ g'(x) &= \frac{x}{x+1} \left(\frac{1}{2\sqrt{x}}\right) \\ g'(x) &= \frac{x}{2\sqrt{x}(x+1)} \end{aligned}$$

7. $y = \int_{\tan x}^{\frac{\pi}{2}} \sin(t^4) dt$

$$\begin{aligned} y' &= \sin\left(\left(\frac{\pi}{2}\right)^4\right)(0) - \sin((\tan x)^4)(\sec^2 x) \\ y' &= -\sin(\tan^4 x)\sec^2 x \end{aligned}$$

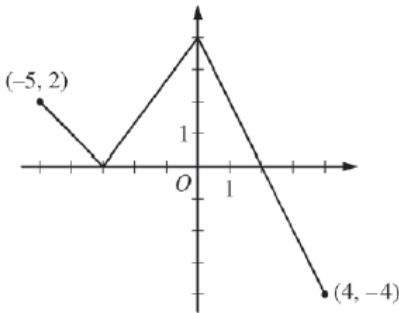
8. $y = \int_{-5}^{\sin x} t \cos(t^3) dt$

$$\begin{aligned} y' &= (\sin x) \cos((\sin x)^3)(\cos x) - (-5) \cos((-5)^3)(0) \\ y' &= \cos(\sin^3 x) \sin x \cos x \end{aligned}$$

Given the graph of f which consists of three line segments as shown on $[-5, 4]$.

Let g be the function given by:

$$g(x) = \int_{-3}^x f(t) dt$$



Determine the following.

Graph of f

- 1.) $g(0) = \int_{-3}^0 f(t) dt = 6$ 2.) $g(2) = \int_{-3}^2 f(t) dt = 10$ 3.) $g(-3) = \int_{-3}^{-3} f(t) dt = 0$
- 4.) $g(4) = \int_{-3}^4 f(t) dt = 6$ 5.) $g(-5) = \int_{-3}^{-5} f(t) dt = -\int_{-5}^{-3} f(t) dt = -2$
6. On what interval(s) of x is g increasing (if any)? $[-5, -3) \cup (-3, 2)$
7. On what interval(s) of x is g decreasing (if any)? $(2, 4]$
8. At what value(s) of x does g have a local maximum (if any)? $x = 2$
9. At what value(s) of x does g have a local minimum (if any)? None
10. At what value of x does g have an absolute maximum? What is the abs. maximum? $x = 2, y = 10$
11. At what value of x does g have an absolute minimum? What is the abs. minimum? $x = -5, y = -2$
12. At what value(s) of x does g have a point of inflection (if any)? $x = -3, 0$
- 13.) $g'(0) = f(0) = 4$ 14.) $g'(2) = f(2) = 0$ 15.) $g'(-4) = f(-4) = 1$
- 16.) $g'(4) = f(4) = -4$
- 17.) $g''(-1) = f'(-1) = \frac{4}{3}$ 18.) $g''(1) = f'(1) = -2$ 19.) $g''(-4) = f'(-4) = -1$
20. $\lim_{x \rightarrow -3} \frac{g(x)}{x^2 - 9} = \lim_{x \rightarrow -3} \frac{\int_{-3}^x f(t) dt}{x^2 - 9} = \frac{0}{(-3)^2 - 9} = \frac{0}{0}$ So now we can apply L'Hopital's Rule
 $\lim_{x \rightarrow -3} \frac{f(x)}{2x} = \frac{f(-3)}{2(-3)} = \frac{0}{-6} = 0$
21. Write the equation of the tangent line to the graph of g .
 - a) at $x = 2$

$$g(2) = \int_{-3}^2 f(t) dt = 10$$

$$g'(2) = f(2) = 0$$

$$y - g(x_1) = g'(x_1)(x - x_1)$$

$$y - 10 = 0(x - 2)$$

$$y = 10$$
 - b) at $x = -5$

$$g(-5) = \int_{-3}^{-5} f(t) dt = -\int_{-5}^{-3} f(t) dt = -2$$

$$g'(-5) = f(-5) = 2$$

$$y - g(x_1) = g'(x_1)(x - x_1)$$

$$y - (-2) = 2(x - (-5))$$

$$y + 2 = 2(x + 5)$$

$$y = 2x + 8$$